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Finite particle creation in 1+1 dim. compact in space

D. J. Lamb*, A. Z. Capri

Theoretical Physics Institute,

Department of Physics, University of Alberta,

Edmonton, Alberta T6G 2J1, Canada

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Abstract

In this paper we calculate the massive particle creation as seen by a stationary observer in a $1+1$ dimensional spacetime compact in space. The Bogolubov transformation relating the annihilation and creation operators between two spacelike surfaces is calculated. The particle creation, as observed by a stationary observer who moves from the first spacelike surface to the second is then calculated, and shown to be finite, as is expected for a spacetime with finite spatial volume.

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I. INTRODUCTION

In the last couple of years there has been renewed interest in the problems associated with defining particles on a curved manifold [1]. Much of this renewed interest shares common ground in the interpretation of the "particles" detected by Rindler observers. The standard analysis as outlined in Birrell and Davies [2] involves relating the annihilation and creation operators of two different quantizations of a field. One quantization is based on the entire spacetime while the other is based on coordinates which only cover the Rindler wedge. As has been shown by Silaev and Krustalev [3], the boundary conditions which one is now forced to impose on the boundaries of the wedge are responsible for the frequency mixing between these modes which is then interpreted as particle creation. Their analysis compares these standard calculations for Rindler observers to those done if one quantizes a field in one half of $1+1$ Minkowski space and compares this to the quantization done over the entire spacetime. Indeed many calculations have been done calculating the Bogolubov transformation relating the operators from two different coordinatisations, one which covers the entire spacetime and one which only covers a portion.

Recently some particle creation calculations have been done by determining how an observer's particle definition evolves with time [4]. The purpose of this paper is to show that the total particle creation for an expanding $1+1$ dimensional spacetime compact in space, is finite. This is in agreement with Fulling's analysis [5] as we are dealing with a finite volume of space. An earlier calculation for an infinite volume of space yielded inconclusive results for the total particle creation which was presumably infinite [6]. This particle creation as interpreted through a nonvanishing $\beta(n,s)$ in the Bogolubov transformation, drops off faster than any inverse power of the momenta n, s which implies that the particle creation if finite and that the Bogolubov transformation is unitarily implementable.

This calculation follow's the procedure of Capri and Roy [7] which is very similar to the procedure advocated by Massacand and Schmid [8]. Both these approaches are based on a coordinate independent approach where the geometry determines the foliation one should

use to quantize the field. This preferred direction of time is given along a normal to the spacelike surface consisting of those spacelike geodesics which are orthogonal to the observers 4-velocity. In this way the construction only depends on the geometry, the observers position, and the tangent to the observer's worldline.

II. THE MODEL

The model we investigate is that of a compact $1+1$ dimensional spacetime described by the metric,

$$ds^2 = dT^2 - e^{\lambda T} R^2 d\theta^2. \qquad 0 \leq \theta < 2\pi \tag{2.1}$$

To follow the prescription mentioned above we first must calculate the geodesics. The first integrals of the geodesic equations are:

$$\frac{d\theta}{ds} = \frac{c_1}{Re^{\lambda T}} \qquad \frac{dT}{ds} = \sqrt{\epsilon + \frac{c_1^2}{e^{\lambda T}}} \tag{2.2}$$

where $\epsilon = \pm 1$ depending on whether the geodesic is timelike or spacelike respectively.

The preferred coordinates on the hypersurface of instantaneity are constructed using a 2-bein of orthogonal basis vectors based at P_0, the observer's position. We choose these vectors to be,

$$e_0(P_0) = (1,0) \quad e_1(P_0) = (0, \frac{1}{Re^{\frac{\lambda T_0}{2}}}), \tag{2.3}$$

in this way $e_0(P_0)$ is tangent to the observer's worldline at P_0.

To construct a spacelike geodesic which is orthogonal to the observer's world line it is required that,

$$\frac{dT}{ds}|_{P_0} = 0 \quad \text{which} \quad \text{implies} \quad \frac{c_1^2}{e^{\lambda T_0}} = 1 \tag{2.4}$$

The preferred coordinates on the spacelike hypersurface are chosen to be Riemann coordinates based on the observer's position $P_0 = (T_0, \theta_0)$. The point $P_1 = (T_1, \theta_1)$ is the point at which a timelike geodesic "dropped" from an arbitrary point $P = (T, \theta)$ intersects the

spacelike hypersurface orthogonally. The Riemann coordinates η^α of the point P_1 are given by,

$$sp^\mu = \eta^\alpha e^\mu_\alpha(P_0) \tag{2.5}$$

where s is the distance along the geodesic $P_0 - P_1$ and p^μ is the tangent vector, at P_0, to the geodesic connecting P_0 to P_1. These equations can be solved for the η^α using the orthogonality of p^μ to $e_0(P_0)$ and the identity $e^\mu_\alpha e_{\beta\mu} = \eta_{\alpha\beta}$ (Minkowski metric) to give,

$$\eta^0 = sp^\mu e^0_\mu(P_0) \qquad \eta^1 = -sp^\mu e^1_\mu(P_0) \tag{2.6}$$

The surface of instantaneity S_0 is just the surface $\eta^0 = 0$ and the preferred spatial coordinate $x^1 = \eta^1$ is,

$$x^1 = s\frac{c_1}{\sqrt{e^{\lambda T_0}}} \tag{2.7}$$

where s is the geodesic distance between the points P_0 and P_1. The direction of time is given by the normal to this spacelike hypersurface. The preferred time coordinate t for the point P is given by the proper distance along the timelike geodesic connecting P to P_1. This timelike geodesic is also determined by (2.2) with $\epsilon = -1$ and a different choice of integration constant, b_1. The condition that the geodesic connecting P to P_1 is normal to the spacelike hypersurfaceat P_1 requires that

$$\sqrt{1 + \frac{b_1^2}{e^{\lambda T_1}}}\sqrt{\frac{c_1^2}{e^{\lambda T_1}} - \frac{c_1^2}{e^{\lambda T_0}}} = \frac{b_1 c_1}{e^{\lambda T_1}} \tag{2.8}$$

The metric can now be calculated in terms of the preferred coordinates (t, x^1) by calculating $(T(t, x^1), \theta(t, x^1))$. To calculate these dependances we use the above equation for x^1 (2.7) and also calculate the change in the coordinate θ along the spacelike and timelike geodesics which connect P_0 to P,

$$R\theta = R\theta_0 + \int_{T_0}^{T_1} dT \frac{c_1}{e^{\lambda T_1}} \left(\frac{c_1^2}{e^{\lambda T}} - \frac{c_1^2}{e^{\lambda T_0}}\right)^{-\frac{1}{2}} + \int_{T_1}^{T} dT' \frac{b_1}{e^{\lambda T'}} \left(1 + \frac{b_1^2}{e^{\lambda T'}}\right)^{-\frac{1}{2}} \tag{2.9}$$

and

$$t = \int_{T_1}^{T} dT' \left(1 + \frac{b_1^2}{e^{\lambda T'}}\right)^{-\frac{1}{2}} \tag{2.10}$$

By performing the above integral for θ^1 and inverting the t integral one is left with the coordinate transformations

$$\begin{aligned} e^{\frac{\lambda}{2}(T-T_0)} &= \sinh(\frac{\lambda t}{2}) + \cosh(\frac{\lambda t}{2})\cos(\frac{\lambda x^1}{2}) \\ \frac{R\lambda}{2}(\theta - \theta_0)e^{\lambda\frac{T}{2}} &= -\cosh(\frac{\lambda t}{2})\sin(\frac{\lambda x^1}{2}) \end{aligned} \tag{2.11}$$

In terms of the preferred coordinates (t, x^i) the metric now has the form,

$$ds^2 = dt^2 - \cosh^2(\frac{\lambda t}{2})(dx^1)^2. \tag{2.12}$$

The range of x^1 is $0 \leq x^1 < \frac{4\pi}{\lambda}$. To write this in a more convenient form we introduce the angular coordinate $\alpha = \frac{\lambda x^1}{2}$ which covers the range $0 \leq \alpha < 2\pi$. In terms of this angular coordinate the metric takes the form

$$ds^2 = dt^2 - \frac{4}{\lambda^2}\cosh^2(\frac{\lambda t}{2})d\alpha^2 \tag{2.13}$$

III. MODES AND INITIAL CONDITIONS

In the coordinates constructed above the minimally coupled massive Klein Gordon equation is,

$$\partial_t^2\phi + \frac{1}{\sqrt{g}}\partial_t(\sqrt{g})\partial_t\phi + \frac{1}{\sqrt{g}}\partial_1(\sqrt{g}g^{11})\partial_1\phi + m^2\phi = 0 \tag{3.1}$$

To quantize a scalar field on the $t = 0$ surface we now define the positive frequency modes. The positive frequency modes are defined as those which satisfy the initial conditions,

$$\phi_k^+(t, \mathbf{x})\mid_{t=0} = A_k(0, \alpha) \quad \text{and} \quad \partial_t(\phi_k^+(t, \alpha))\mid_{t=0} = -i\omega_k(0)A_k(0, \alpha) \tag{3.2}$$

Where $A_k(t, \alpha)$ are the instantaneous eigenmodes of the spatial part of the Laplace-Beltrami operator, and $\omega_k(t)^2$ are the corresponding eigenvalues.

$$\left[\frac{1}{\sqrt{g}}\partial_1\left(\sqrt{g}g^{11}\partial_1\right)+m^2\right]A_k(t,\alpha)=\omega_k^2(t)A_k(t,\alpha) \tag{3.3}$$

Henceforth we just write ω_k for $\omega_k(0)$. Due to the simple form of $g_{\mu\nu}$ at $t=0$ the eigenmodes and eigenvalues take on the simple form,

$$\begin{aligned} A_k(0,\alpha) &= e^{i\frac{2k}{\lambda}\alpha} \\ \omega_k^2(0) &= k^2+m^2. \end{aligned} \tag{3.4}$$

Near the surface $t=0$ the second term of (3.1) vanishes to $O(t^2)$, this implies that the initial conditions for the time dependence of the field are also good to $O(t^2)$. We impose periodic boundary conditions on $A_k(0,\alpha)$ to choose a self adjoint extension for the differential operator on the left hand side of (3.3). This requires that $\frac{2k}{\lambda}=s$ where s is an integer.

To impose the initial conditions we need a complete set of modes for the entire wave operator. Because the wave equation is separable we look for solutions of the form $f_s(t)e^{is\alpha}$. The differential equation satisfied by the $f_s(t)$ is then,

$$\partial_t^2 f_s(t)+\frac{\lambda}{2}\tanh(\frac{\lambda t}{2})\partial_t f_s(t)+\left(\frac{s^2\lambda^2}{4}\mathrm{sech}^2(\frac{\lambda t}{2})+m^2\right)f_s(t)=0 \tag{3.5}$$

The positive frequency modes are those whose time part satisfies the above differential equation and the initial conditions,

$$f_s(0)=1 \quad \text{and} \quad \dot{f}_s(0)=-i\omega_k. \tag{3.6}$$

The positive frequency modes are given in terms of hypergeometric functions $H(a,b,c,g(t))$ by

$$\phi_s^+(t,\alpha)=e^{is\alpha}\cosh(\frac{\lambda t}{2})^s\left\{\, H(\alpha,\beta,\frac{1}{2},-\sinh^2(\frac{\lambda t}{2}))-i\frac{2\omega_s}{\lambda}\sinh(\frac{\lambda t}{2})H(\alpha+\frac{1}{2},\beta+\frac{1}{2},\frac{3}{2},-\sinh^2(\frac{\lambda t}{2}))\right\} \tag{3.7}$$

where

$$\alpha=\frac{s}{2}+\frac{1}{4}+\frac{i}{4\lambda}\sqrt{16m^2-\lambda^2}$$

$$\beta = \frac{s}{2} + \frac{1}{4} - \frac{i}{4\lambda}\sqrt{16m^2 - \lambda^2}$$
$$\omega_s = \sqrt{(\frac{\lambda s}{2})^2 + m^2}. \tag{3.8}$$

We can now write out the field which has been quantized on surface 1 as,

$$\Psi_1 = \sum_{s=-\infty}^{s=\infty} \frac{1}{\sqrt{2\omega_s}} \left\{ \phi_s^+(t,\alpha) a_1(s) + \phi_s^{+*}(t,\alpha) a_1^\dagger(s) \right\} \tag{3.9}$$

IV. PARTICLE CREATION

To investigate particle creation in the model universe as observed by an observer stationary with respect to the original coordinates (T, θ) we calculate the Bogolubov transformation relating the annihilation and creation operators from two different surfaces of quantization that the observer passes through. To calculate the coefficients of this transformation we equate the same field from two different quantizations on a common surface,

$$\Psi_1(t,\alpha) = \Psi_2(t'(t,\alpha), \alpha'(t,\alpha)). \tag{4.1}$$

Here $\Psi_1(t,\alpha)$ is the field written out explicitly in (3.9) and $\Psi_2(t',\alpha')$ is the same field which has been quantized on a second surface $t' = 0$. The "second" field is therefore quantized for the same observer as the first but at some later time T'_0 with $\theta_0 = \theta'_0$. All the physics of the observations made by this observer are determined by the functions $t'(t,\alpha)$, $x'(t,\alpha)$ and the derivatives of these functions with respect to t. In this way the geometry of the spacetime via the coordinate independent prescription we have used, determines the spectrum of created particles.

We calculate the Bogolubov transformation by "matching" the field and its first derivative with respect to t at $t = 0$.

$$a_1(s) = \frac{i}{(2\pi)} \frac{1}{\sqrt{2\omega_s}} \int_0^{2\pi} d\alpha e^{-is\alpha} \left\{ i\omega_s \Psi_1(0,x) - (\partial_t \Psi_1(t,\alpha))\,|_{t=0} \right\}$$
$$= \frac{i}{(2\pi)} \frac{1}{\sqrt{2\omega_s}} \int d\alpha e^{-is\alpha} \left\{ i\omega_s \Psi_2(t'(0,\alpha), \alpha'(0,\alpha)) - (\partial_t \Psi_1(t'(t,\alpha), \alpha'(t,\alpha)))\,|_{t=0} \right\} \tag{4.2}$$

Using this equation, we can write out the Bogolubov transformation in the form

$$a_2(n) = \sum_s \alpha(n,s) a_1(s) + \beta(n,s) a_1^\dagger(s). \tag{4.3}$$

The spectrum of created particles is determined by $|\beta(n,s)|^2$.

Writing out $\beta(n,s)$ explicitly we find it has some interesting properties due to it's dependence on the inverse relations $t'(t,x)$,$x'(t,x)$,

$$\beta(n,s) = \frac{-i}{2\pi} \int d\alpha \frac{e^{-in\alpha}}{\sqrt{4\omega_n \omega_s}} \left\{ i\omega_n f_s^{+*}(t'(0,\alpha)) e^{-is\alpha'(0,\alpha)} - \partial_t \left\{ f_s^{+*}(t'(t,\alpha)) e^{-is\alpha'(t,\alpha)} \right\} |_{t=0} \right\} \tag{4.4}$$

where

$$\begin{aligned} \alpha'(t,\alpha) &= \tan^{-1}\left(\frac{\cosh(\frac{\lambda t}{2}) \sin(\alpha)}{\cosh(\frac{\lambda t}{2}) \cos(\alpha) \cosh(\tau) - \sinh(\frac{\lambda t}{2}) \sinh(\tau)} \right) \\ t'(t,\alpha) &= \frac{2}{\lambda} \sinh^{-1}\left(\sinh(\frac{\lambda t}{2}) \cosh(\tau) - \cosh(\frac{\lambda t}{2}) \cos(\alpha) \sinh(\tau) \right) \end{aligned} \tag{4.5}$$

and

$$\tau = \frac{\lambda}{2}(T_0' - T_0) \tag{4.6}$$

V. TOTAL NUMBER OF PARTICLES CREATED

To find out whether the total number of particles created is finite we must find out if $\beta(n,s)$ is Hilbert-Schmidt, namely

$$\sum_s \sum_n |\beta(n,s)|^2 < \infty. \tag{5.1}$$

If this inequality holds it means that the total number of created particles is finite and the Bogolubov transformation is unitarily implementable. To calculate the total number of particles created we write $\beta(n,s)$ in a slightly different form,

$$\beta(n,s) = \frac{-i}{4\pi\sqrt{\omega_s \omega_n}} \int d\alpha e^{-i(n+s)\alpha} e^{-is(\alpha'(0,\alpha)-\alpha)} g(n,s,\alpha) \tag{5.2}$$

where

$$g(n,s,\alpha) = \left\{ i\omega_n f_s^{+*}(t'(0,\alpha)) - is\frac{\partial\alpha'}{\partial t} f_s^{+*}(t'(t,\alpha)) - \frac{\partial t'}{\partial t}\partial_{t'}\left(f_s^{+*}(t'(t,\alpha))\right)\right\}|_{t=0} \tag{5.3}$$

We have written $\beta(n,s)$ is this form allow us to write $\alpha'(0,\alpha)$ in a form which takes care of the problem of which branch of the $\tan^{-1}(y)$ in (4.5) to take.

To investigate the asymptotic form of $g(n,s,\alpha)$ we have to find the asymptotic behaviour of the hypergeometric functions involved. The first simplification that can be made is due to the fact that $\beta = \alpha^*$. By writing $\alpha = a + ib$ we see directly from the series for the hypergeometric functions that for large a one can drop the imaginary part of α

$$\begin{aligned} H(\alpha,\beta,c,z) &= 1 + \frac{\alpha\beta}{c}z + \frac{\alpha\beta(\alpha+1)(\beta+1)}{c(c+1)}\frac{z^2}{2} + ... \\ &= 1 + \frac{(a^2+b^2)}{c}z + \frac{(a^2+b^2)((a+1)^2+b^2)}{c(c+1)}\frac{z^2}{2} + ... \\ &\approx 1 + \frac{(a^2)}{c}z + \frac{(a^2)(a+1)^2}{c(c+1)}\frac{z^2}{2} + ... \\ &= H(a,a,c,z) \end{aligned} \tag{5.4}$$

From (3.7) we see that we need asymptotic forms for hypergeometric functions of the form $H(a,a,\frac{1}{2},-x^2)$ and $xH(a+\frac{1}{2},b+\frac{1}{2},\frac{3}{2},-x^2)$. For the first form we can write the hypergeometric function in terms of a Legendre function using the identity [9]

$$H = (a,a,1/2,-x) = \frac{\pi^{\frac{-1}{2}}}{2}\Gamma(a+\frac{1}{2})\Gamma(1-a)(1+x)^{-a}\left(P_{2a-1}[\frac{x^{\frac{1}{2}}}{\sqrt{(1+x)}}] + P_{2a-1}[\frac{-x^{\frac{1}{2}}}{\sqrt{(1+x)}}]\right) \tag{5.5}$$

To obtain the asymptotic form for $xH(a+\frac{1}{2},b+\frac{1}{2},3/2,-x^2)$ we notice that we can write it in terms of the derivative of the first hypergeometric function,

$$xH(a+\frac{1}{2},a+\frac{1}{2},\frac{3}{2},-x^2) = \frac{-1}{2(a-\frac{1}{2})^2}\frac{d}{dx}H(a-\frac{1}{2},a-\frac{1}{2},1/2,-x^2) \tag{5.6}$$

We now use an expression for the Legendre functions valid for large ν [10],

$$P_\nu[\cos(\theta)] \approx \frac{\Gamma(\nu+1)}{\Gamma(\nu+\frac{3}{2})}\sqrt{\frac{2}{\pi\sin(\theta)}}\cos((\nu+\frac{1}{2})\theta - \frac{\pi}{4}) \tag{5.7}$$

By using the reflection formula

$$\Gamma(1-x) = \frac{\pi}{\Gamma(x)\sin(\pi z)} \tag{5.8}$$

and taking the asymptotic form for the gamma functions which is valid for large argument,

$$\Gamma(ax+b) \approx \sqrt{2\pi}e^{-ax}(ax)^{ax+b-\frac{1}{2}} \tag{5.9}$$

we find that the gamma functions from (5.5) and (5.7) combine in such a way as to cancel their s dependence leaving,

$$\begin{aligned}
\beta(n,s) \quad &= \frac{1}{4\pi\sqrt{\omega_s\omega_n}}\int d\alpha e^{-i(n+s)\alpha}e^{-is(\alpha'(0,\alpha)-\alpha)}\left(\cos[s\cos^{-1}[p(\alpha)]] + \sin[s\cos^{-1}[p(\alpha)]]\right) \\
&\times\left(A(\alpha)\left(\cos[\frac{\pi s}{2}] + \sin[\frac{\pi s}{2}]\right) + B(\alpha)\left(\cos[\frac{\pi s}{2}] - \sin[\frac{\pi s}{2}]\right)\right)
\end{aligned} \tag{5.10}$$

where

$$p(\alpha) = \frac{\cos(\alpha)\sinh(\tau)}{\sqrt{1+\cos^2(\alpha)\sinh^2(\tau)}} \tag{5.11}$$

$$\begin{aligned}
A(\alpha) = &\frac{\lambda\,|s|}{f(\alpha)^2(2\,|s|-1)^2}\left\{f(\alpha)^2\,|n|\,(1-(-1)^s)(|s|-1) + i\cos[\alpha]\sinh[\tau](1-(-1)^s)\right. \\
&+ i\cosh[\tau](1+(-1)^s)(2-f(\alpha)+f(\alpha)\,|s| - f(\alpha)s^2) \\
&\left.+ \sin[\alpha]\sinh[\tau](1-(-1)^s)(f(\alpha)s\,|s| - f(\alpha)s)\right\}
\end{aligned} \tag{5.12}$$

$$B(\alpha) = \frac{\lambda}{4f(\alpha)}\left\{if(\alpha)\,|n|\,(1+(-1)^s)) + |s|\cosh[\tau](1-(-1)^s) + is\sin[\alpha]\sinh[\tau](1+(-1)^s)\right\} \tag{5.13}$$

and

$$f(\alpha) = 1 + \cos^2[\alpha]\sinh^2[\tau] \tag{5.14}$$

The entire point of writing $\beta(n,s)$ in this way was to allow us to integrate the above expression by parts. After expanding the $\sin[s\cos^{-1}[p(\alpha)]]$ and $\cos[s\cos^{-1}[p(\alpha)]]$ in terms of exponentials, each term making up $\beta(n,s)$ can be written in the form,

$$\beta(n,s) \propto \int d\alpha e^{-i(n+s)\alpha} e^{isg(\alpha)} K_{n,s}(\alpha) \tag{5.15}$$

where $K_{n,s}(\alpha)$ incorporates the last term in (5.10) which contains $A(\alpha)$ and $B(\alpha)$ and

$$g(\alpha) = -(\alpha'(0,\alpha) - \alpha) \pm \cos^{-1}(p(\alpha)) \tag{5.16}$$

where the $\pm$ depends on which of the two terms one is dealing with. The important point is that the behaviour of $K_{n,s}(\alpha)$ in terms of n, s does not change because the dependence on n, s is decoupled from α. not increase if one differentiates it with respect to α. One can now integrate by parts indefinitely to observe that the expression must drop off faster than any inverse power of n, s. For example after integrating by parts twice one is left with,

$$\beta(n,s) \propto \int d\alpha e^{-i(n+s)\alpha} e^{\pm isg(\alpha)} \frac{d}{d\alpha}\left(\frac{1}{-i(n+s) \pm g'(\alpha)} \frac{d}{d\alpha}\left(\frac{K_{n,s}(\alpha)}{-i(n+s) \pm g'(\alpha)}\right)\right) \tag{5.17}$$

The only problem that could arise is if $-(n+s) - g'(\alpha)$ ever vanished. This is not a problem however because we are interested in the large momenta limit and $g'(\alpha)$ is a well behaved function.

VI. CONCLUSIONS

We have shown that $\beta(n,s)$ drops off faster than any inverse power of n, s, for large n, s. This implies that the total number of particles created is finite and therefore the Bogolubov transformation is unitarily implementable. The fact that the total number of particles created is finite is in agreement with Fulling's analysis for an isotropic universe of finite spatial volume [5].

If in fact $\beta(n,s)$ drops off like an exponential then after performing one of the sums in $|\beta(n,s)|^2$ one will be left with a Planck spectrum. This is to be expected as for large momenta our analysis should be similar in nature to the analysis of massless particle creation.

It should be emphasized that this calculation does not involve calculating the Bogolubov transformation relating in essence to different spacetimes. This calculation involves comparing an observer's particle definition at two different times in the same spacetime. In this

way one is not misinterpreting boundary effects by comparing fields quantized in overlapping but different spacetimes [3] as is the case in the standard Rindler analysis and many other calculations.

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